The shape of quantum fluctuations

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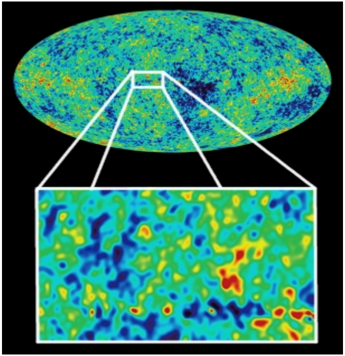


Real space

- 2-point correlations
- Minkowski Functionals
- Extrema counts
- ...
- Tensor Minkowski Functionals

Harmonic space

- Power spectrum
- Bi-tri-spectrum, ...
- Wavelets
- ...

Outline

- Introduction to Tensor Minkowski Functionals (TMF)
- TMFs for the CMB
  - Shape and relative alignment of structures
  - Distribution of shapes
- Application to PLANCK data

References

Mathematical foundations:

McMullen (1997), Alesker (1999), Hug *et al.* (2008)

Physical applications

Beisbart, Valdernini & Buchert (2001)
Beisbart *et al.* (2002)
Schroeder-Turk, Kapfer *et al.* (2009)
Schroeder-Turk, Mickel *et al.* (2013)

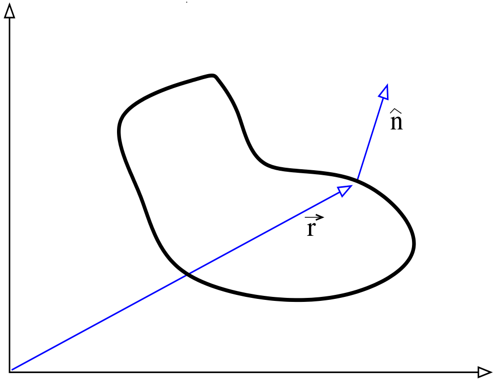
Many applications to systems in biology, chemistry, geology, etc.

Cosmological applications

Vidhya G. & P. Chingangbam (2016)

P. Chingangbam, Vidhya G., S. Appleby, C. Park, *in progress*

Tensor Minkowski Functionals - 2D space



$$W_0^{m,n} = \int \vec{r}^{\,m} \otimes \hat{n}^n \; \mathrm{d}a$$

$$W_1^{m,n} = \int_C \vec{r}^{\,m} \otimes \hat{n}^n \; \mathrm{d}\ell$$

$$W_2^{m,n} = \int_C \vec{r}^{\,m} \otimes \hat{n}^n \, \kappa \; \mathrm{d}\ell$$

$\kappa \equiv$ local curvature

$$\left(\vec{x} \otimes \vec{y}\right)_{ij} \equiv \frac{1}{2}\left(x_i y_j + x_j y_i\right)$$

$$m + n \leq 2$$

Scalar Minkowski Functionals : $m = 0,\ n = 0$

$$W_0 = \int \mathrm{d}a \longrightarrow \text{area}$$

$$W_1 = \int_C \mathrm{d}\ell \longrightarrow \text{contour length}$$

$$W_2 = \int_C \kappa\ \mathrm{d}\ell \longrightarrow \text{genus}$$

Vector Minkowski Functionals : $(m,\ n) = (1,0),\ (0,1)$

$$W_0^{1,0}, \quad \cancel{W_0^{0,1}}, \quad W_1^{1,0}, \quad \cancel{W_1^{0,1}}, \quad W_2^{1,0}, \quad \cancel{W_2^{0,1}}$$

Beisbart, Valdernini & Buchert *et al.* (2001)

Tensor Minkowski Functionals: $(m,\ n) = (1,1), (2,0), (0,2)$

$$\begin{array}{ccc} W_0^{1,1}, & \underbrace{W_0^{2,0}}, & \cancel{W_0^{0,2}} \\ W_1^{1,1}, & \underbrace{W_1^{2,0}}, & W_1^{0,2} \\ W_2^{1,1}, & \underbrace{W_2^{2,0}}, & W_2^{0,2} \end{array}$$

Of these only four are linearly independent. Divides into translation invariant and translation covariant tensors.

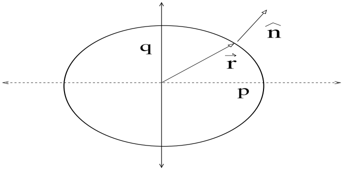
Shape measures based on translation covariant tensors not mathematically developed yet. Can be useful for position dependent information about structures.

Focus on

$$W_2^{1,1} = \int_C \vec{r} \otimes \hat{n} \ \kappa \ \ \mathrm{d}\ell$$

Translation invariant : choice of origin does not matter.

Example: ellipse



$$W_2^{1,1} = \begin{bmatrix} f(p,q) & 0 \\ 0 & f(q,p) \end{bmatrix}$$

$$f(p,q) = \frac{1}{2} p^2 q^2 \int_0^{2\pi} d\varphi \frac{cos^2\varphi}{\left[p^2 - (p^2 - q^2) cos^2\varphi\right]^{3/2}}$$

Measure of intrinsic anisotropy of structures

Single structure:

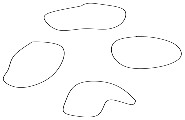
$$W_2^{1,1} \longrightarrow \lambda_1, \lambda_2$$

$$\beta \equiv \frac{\lambda_1}{\lambda_2}, \quad \lambda_1 < \lambda_2$$

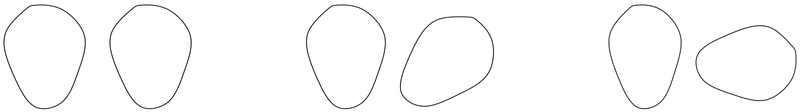
$$0 \leq \beta \leq 1$$

Many structures: average anisotropy

$$\beta \equiv \left\langle \frac{\lambda_1}{\lambda_2} \right\rangle$$



Measure of relative orientation of many structures



Average over all structures $\longrightarrow$ $\left\langle W_2^{1,1} \right\rangle$ $\longrightarrow$ Λ_1, Λ_2

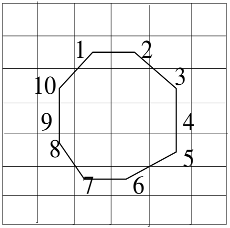
$$\alpha \equiv \frac{\Lambda_1}{\Lambda_2}$$

$$0 \leq \alpha \leq 1$$

Numerical calculation of $W_2^{1,1}$

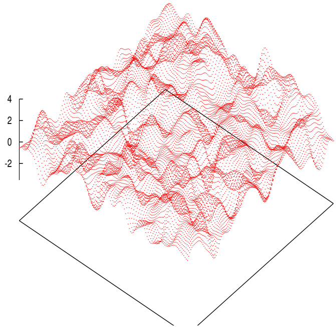
Schroeder-Turk, Kapfer *et al.* (2009)

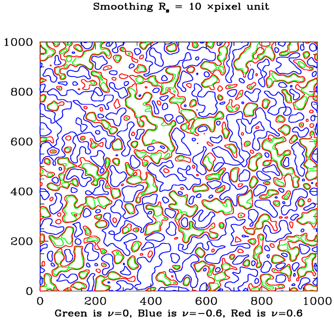
- Space is pixelized.
- Continuous curves are approximated by polygons



$$W_2^{1,1} = \frac{1}{2} \sum_{(i,j)} \frac{1}{|e_{ij}|} \left(\vec{e}_{ij} \otimes \vec{e}_{ij} \right)$$

Excursion sets of random fields





- CMB fields given on surface of sphere.
- Use stereographic projection to project each hemisphere onto circular disk on the 2D plane.
- Shape of curves not changed. Size gets scaled.

β and α for ΛCDM cosmology

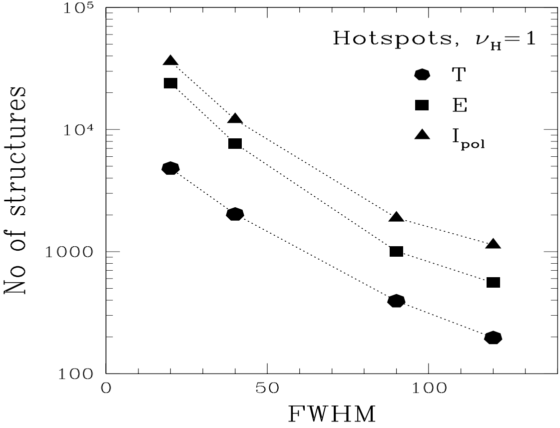
Expectations

- Average β less than 1.
- Isotropic primordial power spectrum

$$\downarrow$$

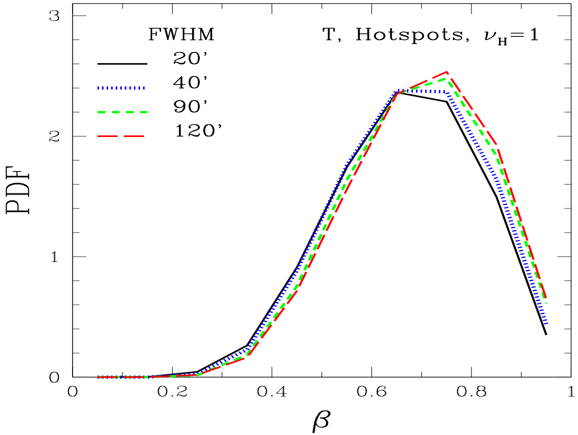
Expect to recover completely unaligned structures, $\alpha = 1$.

β for ΛCDM cosmology



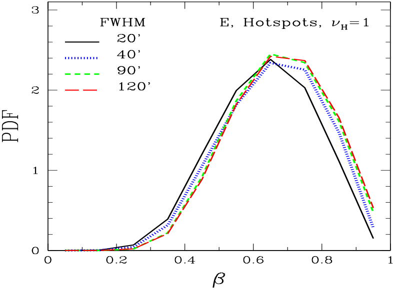
No of structures scales as θ_s^{-2}.

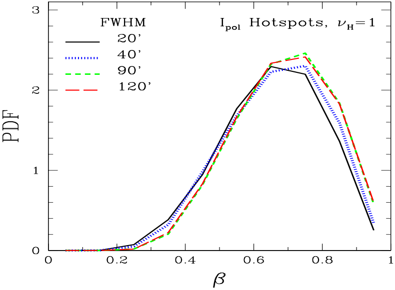
β for ΛCDM cosmology: temperature

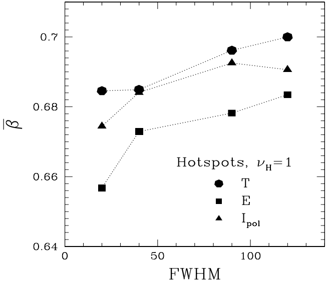


Larger smoothing pushes structures towards more isotropic shape.

β for ΛCDM cosmology: polarization

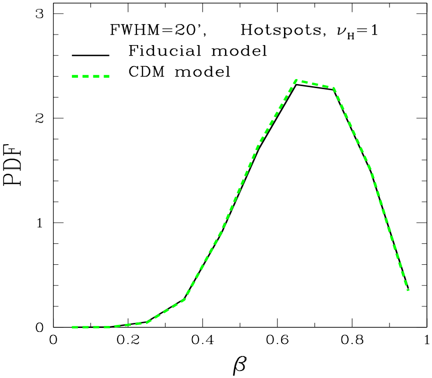






Dependence on Cosmological parameters

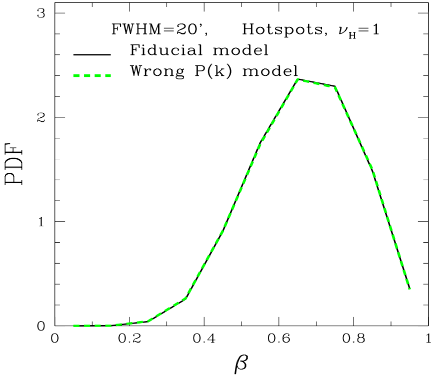
$$\Omega_m \;=\; 0.95$$



β is not very sensitive to variation of matter content.

Dependence on primordial power spectrum

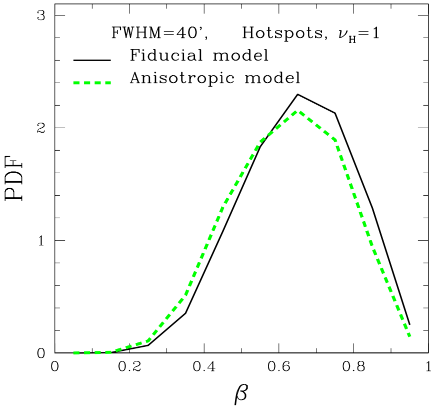
$$P(k) \sim \left(\frac{k}{k_0}\right)^{n_s-1}, \quad n_s = 0.65$$



Change of $P(k)$ changes the relative number of large and scale structures. But β is unaffected.

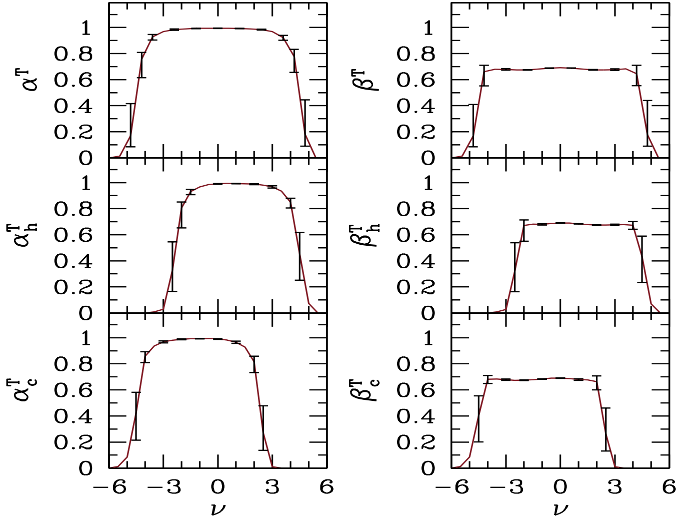
Dependence on statistical anisotropy

$$T(\hat{n}) \;=\; T^{\mathrm{iso}}(\hat{n})\;(1\;+\;A\;\hat{n}\cdot\hat{p}),\quad A\;=\;0.2$$



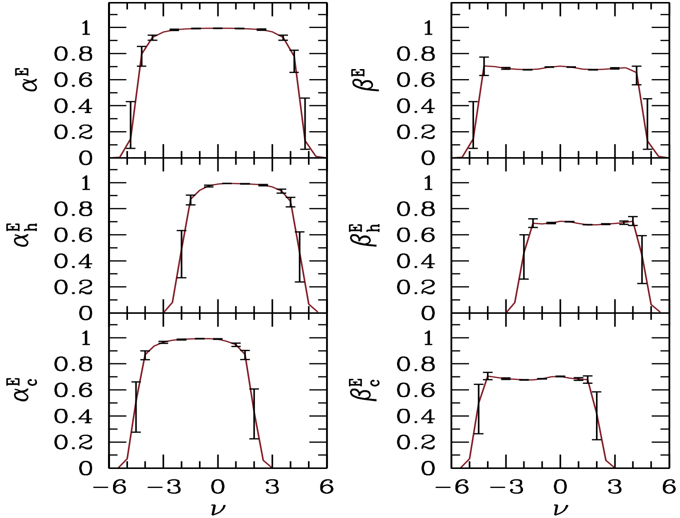
β is sensitive to hemispherical anisotropy. Structures become more anisotropic.

α and average β for ΛCDM cosmology: T



FWHM=20', average over 100 maps

α and average β for ΛCDM cosmology: E mode



FWHM=40', average over 100 maps

Summary of simulation results

- ΛCDM predicts average $\beta \sim 0.68$.
- $\alpha \sim 1$ is recovered, as expected.
- Distribution of β is relatively unaffected by variation of matter content and the primordial power spectrum.
- Distribution of β is sensitive to hemispherical anisotropy.
- Potentially useful for constraining physical effects that introduce anisotropy - anisotropic cosmological models, non-linearity, foregrounds, instrumental effects such as beam shapes, etc.

Application to PLANCK 2015 data

Hotspots: $\nu = 1$, Coldspots: $\nu = -1$
FWHM=20' for T, 40' for E mode.

Field and structure	Planck data		Average from 100 simulations	
	α	β	$\overline{\alpha}$	$\overline{\beta}$
T hotspot	0.9889	0.6795	$0.9911^{+0.0034}_{-0.0054}$	$0.6754^{+0.0026}_{-0.0030}$
T coldspot	0.9936	0.6791	$0.9910^{+0.0038}_{-0.0052}$	$0.6754^{+0.0030}_{-0.0026}$
E mode hotspot	0.9673	0.6820	$0.9930^{+0.0034}_{-0.0034}$	$0.6858^{+0.0022}_{-0.0028}$
E mode coldspot	0.9593	0.6812	$0.9928^{+0.0028}_{-0.0038}$	$0.6854^{+0.0032}_{-0.0030}$

Findings

β

- Both temperature and E data from PLANCK agrees with the simulation results within 3-σ.

α

- Temperature data from PLANCK agrees with the simulation results within 3-σ.

- E mode data from PLANCK gives $\alpha \sim 0.96$. Deviates from isotropic distribution of structures at 14-σ.

Future prospects

- Further investigations for the CMB on how to constrain cosmology.
- Constraining reionization history using 21cm brightness temperature.
- Large scale structure application - probing nonlinear gravitational physics.

Integrand of Hypergeometric function

$$f(\beta) = A\frac{\big(\beta(1-\beta)\big)^{\ell}}{(1-z\beta)^{\ell+1/2+ns/4}}$$

$$0 < z < 1$$

n_s is the primordial spectral index. $f(\beta)$ gets more localized as ℓ increases. Peak shifts towards one as z moves towards one.

Fitting $f(\beta)$ to the PDF seems to give characteristic scale determined by the best fit ℓ and z values.